Optimization of a purlin punching process

Jan Pöschko

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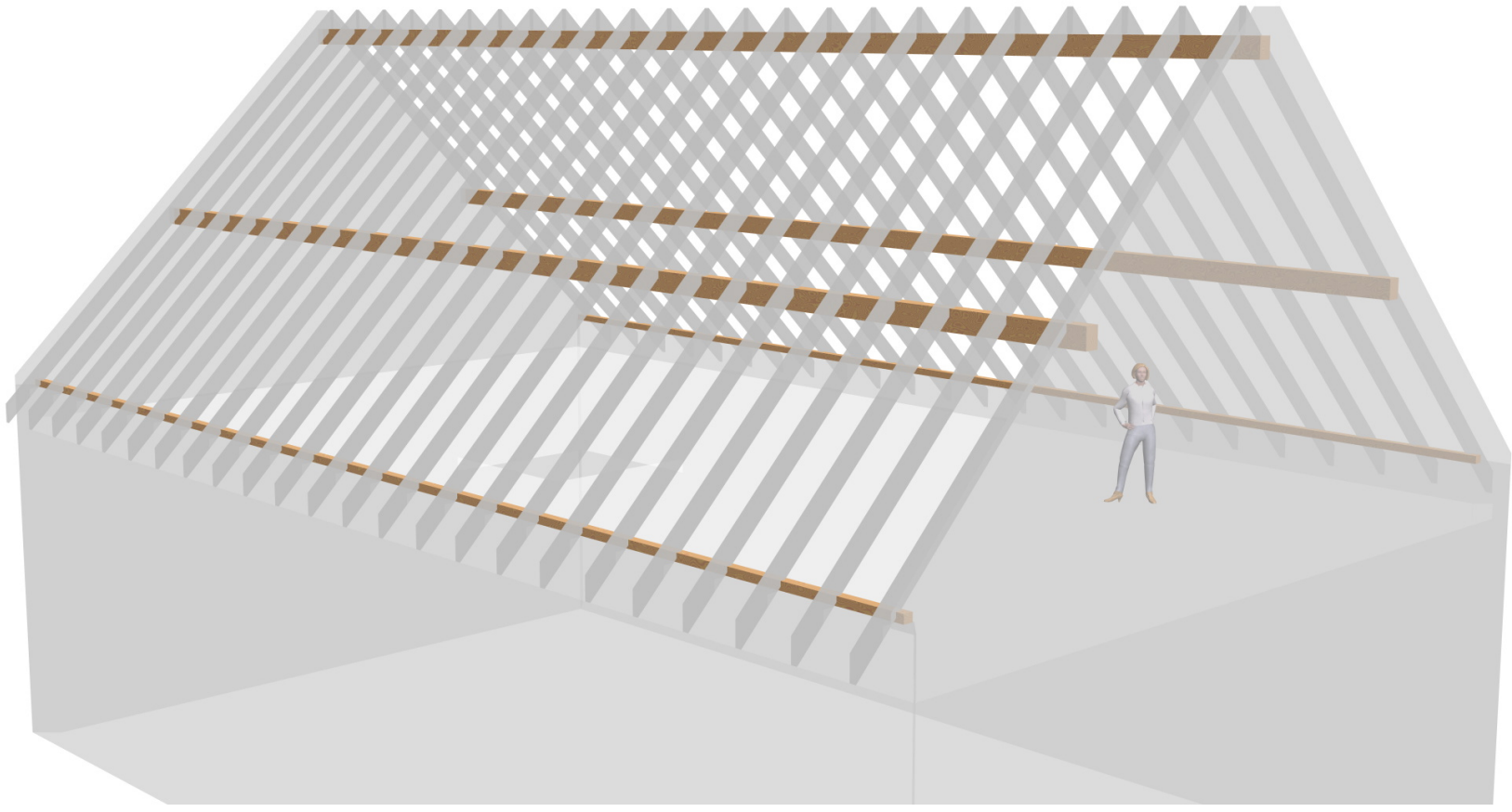
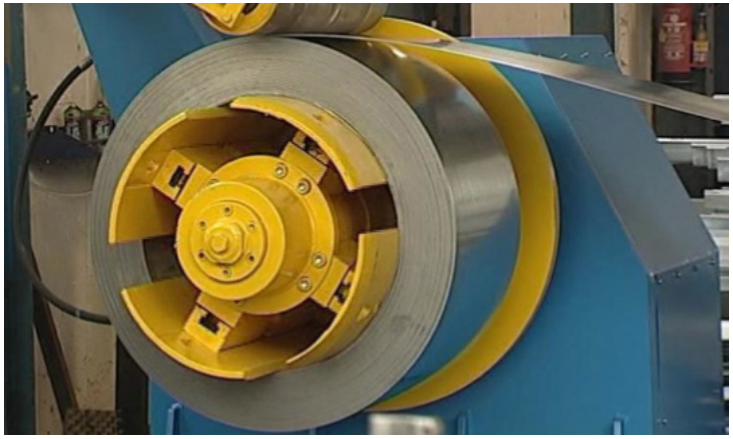
Purlins



(http://de.wikipedia.org/w/index.php?title=Datei:Pfetten_in_Pfettendach.jpg)

Production

1. sheet of metal runs from coil



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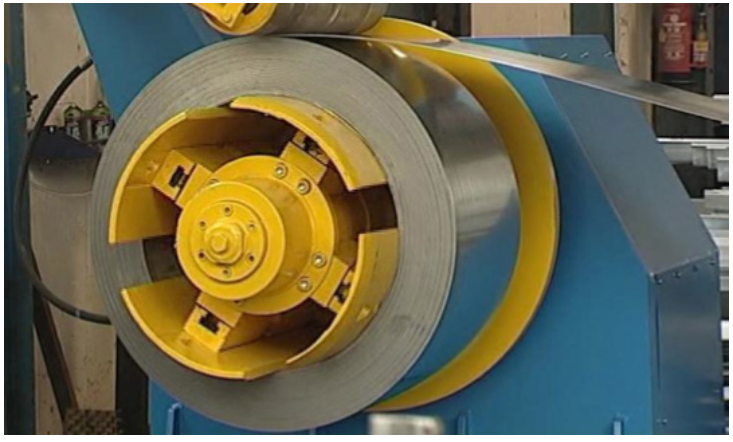

2. holes are punched

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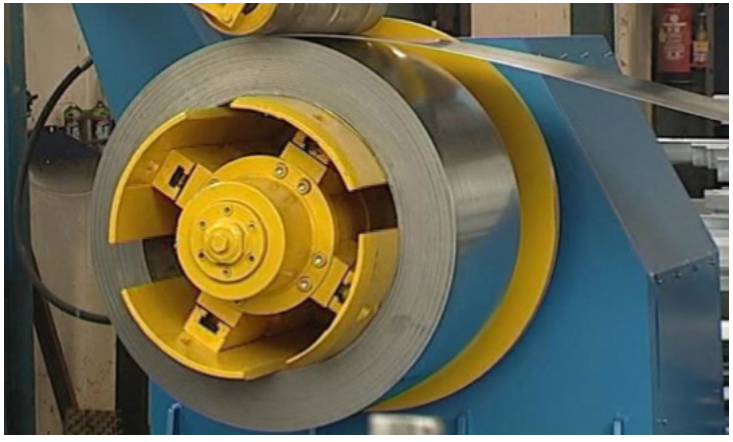

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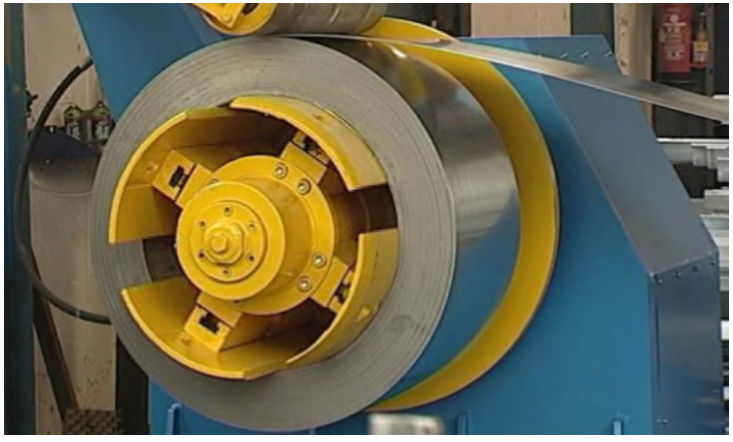

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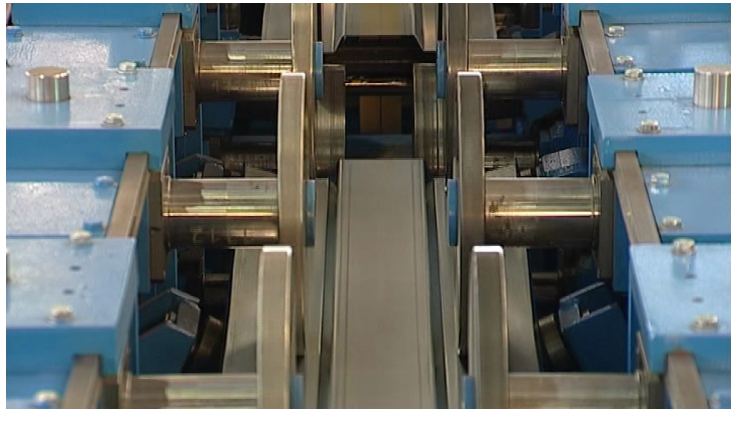
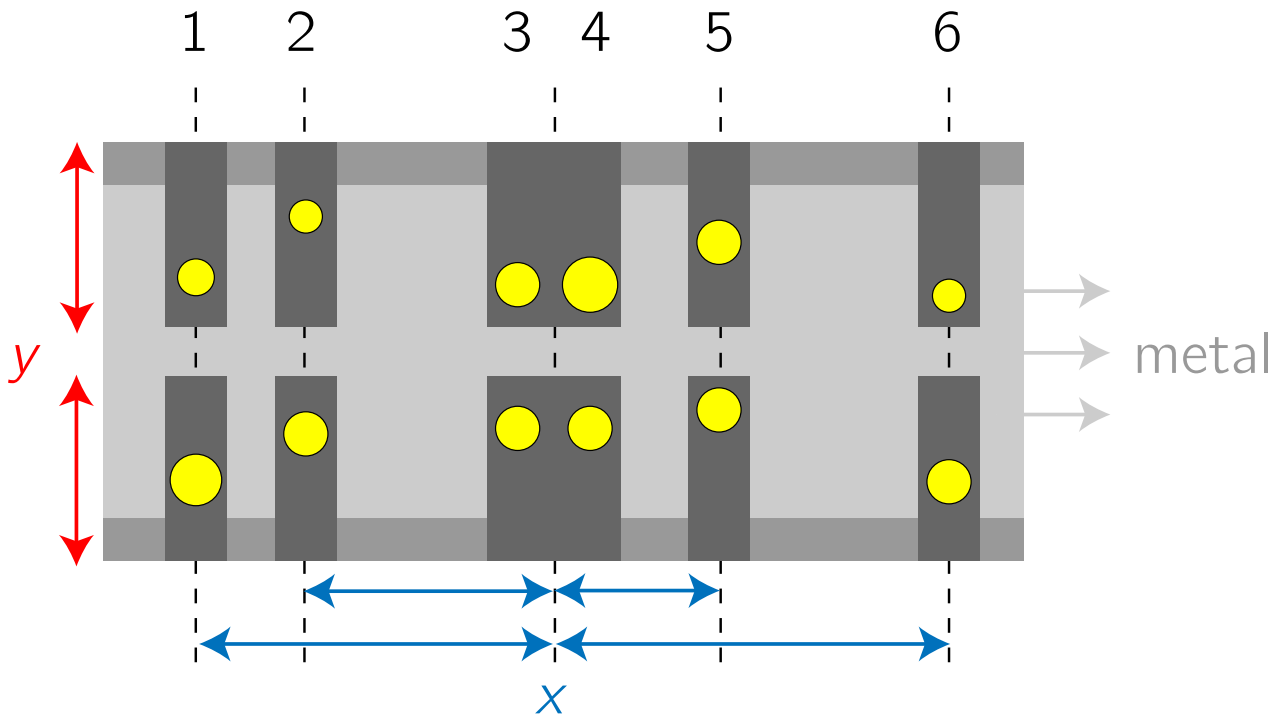
4. metal is folded



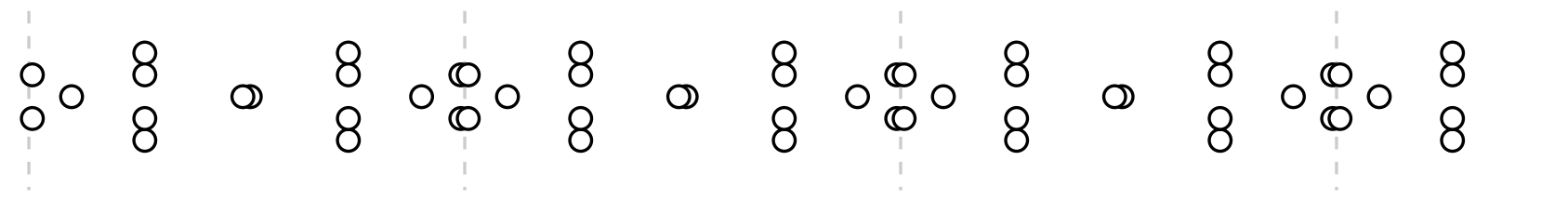
Punching machine

- 6 pairs of stamps
- constant assignment of tools (diameters/shapes) to stamps
- x-positions dynamic with restrictions
- y-positions fully dynamic



Optimization problem

given holes by $x_n \in \mathbb{R}$, $t_n \in T \subset \mathbb{Z}$

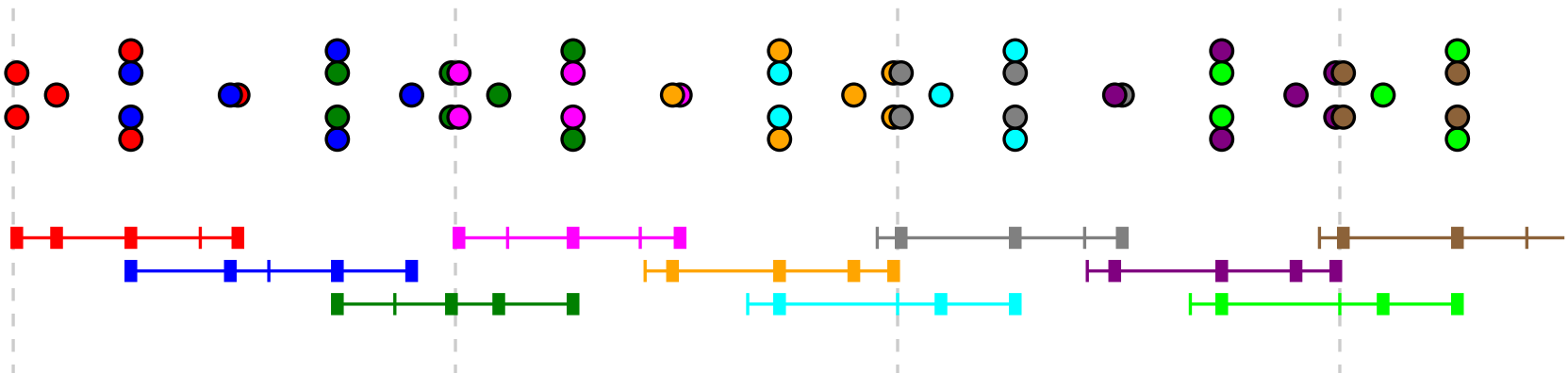


Optimization problem

given holes by $x_n \in \mathbb{R}$, $t_n \in T \subset \mathbb{Z}$
find

- equipping of the machine: $e_j \in T$, $j \in S = \{1, \ldots, 6\}$
- subsequent positions of stamps and punched holes:

$$s_{ij} \in \mathbb{R}, \quad h_{ijn} = \begin{cases} 1 & \text{hole } n \text{ punched by stamp } j \text{ in turn } i \\ 0 & \text{otherwise} \end{cases}$$



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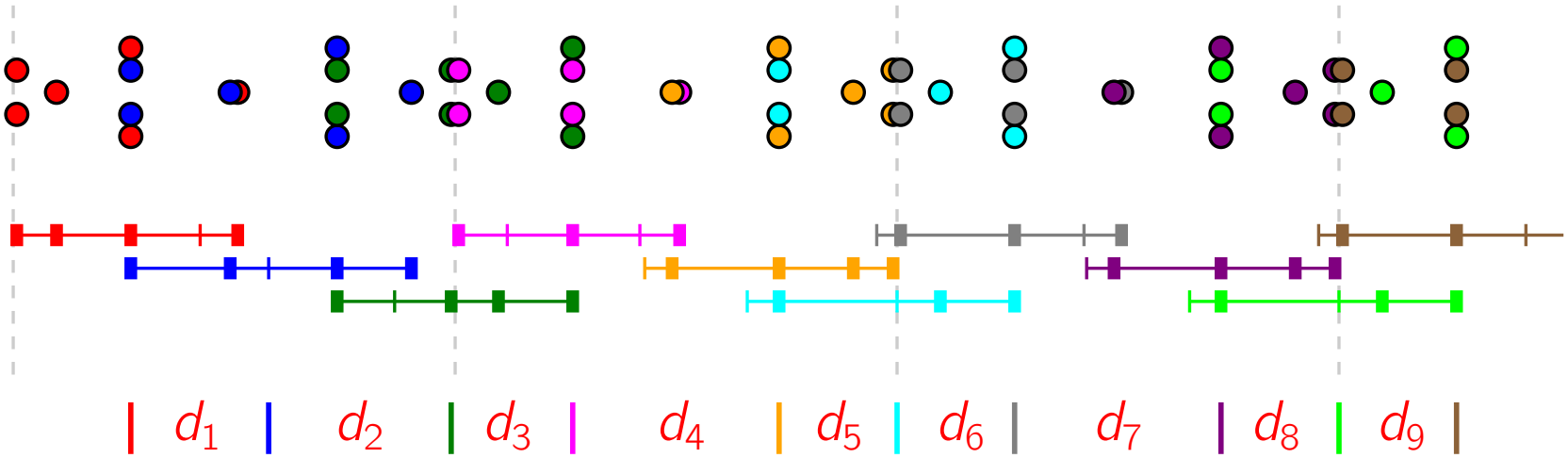
maximizing displacements $d_i = s_{i,4} - s_{i-1,4}$



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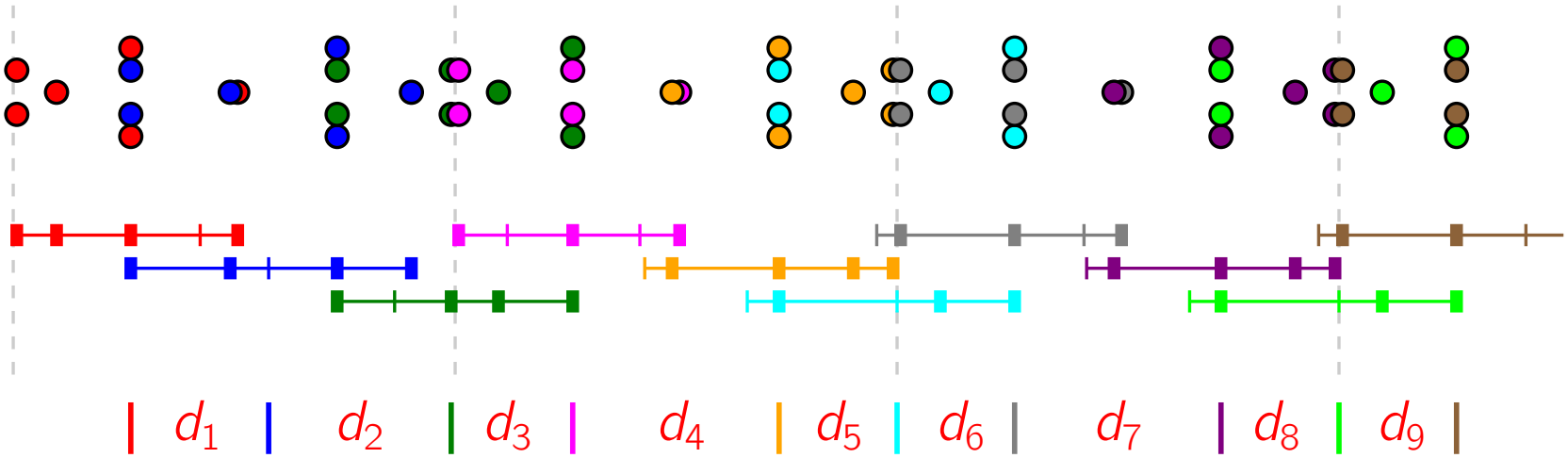

Program formulation

$$\min \sum \frac{1}{d_i}$$

s. t.

$$\sum_n t_n h_{ijn} = e_j \quad \forall i \; \forall j \in A_i \quad \text{(active stamps have correct tools)}$$

$$\sum_n x_n h_{ijn} = s_{ij} \quad \forall i \; \forall j \in A_i \quad \text{(active stamps are on correct positions)}$$

$$\sum_i \sum_{j \in A_i} h_{ijn} = 1 \quad \forall n \quad \text{(each hole is punched exactly once)}$$

where $A_i = \{j \mid \exists n : h_{ijn} = 1\}$ is the set of active stamps in turn i

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where $A_i = \{j \mid \exists n : h_{ijn} = 1\}$ is the set of active stamps in turn i and machine constraints

$$c_j \leq \overline{s}_{ij} - \overline{s}_{i,j-1} \leq C_j \quad \text{(distances between stamps are feasible)}$$

$$|\overline{s}_{ij} - \overline{s}_{i-1,j}| \leq C \quad (x\text{-movement is not too large})$$

where $\overline{s}_{ij} = s_{ij} - s_{i,4}$ is the relative stamp position

Algorithm: determining a feasible starting solution

for all feasible equippings $E = (e_j) \in T^6$ of the machine **do**
  for random fixed relative stamp positions $S = (\bar{s}_j) \in \mathbb{R}^6$ **do**
    determine a punching plan P_{ES} using a greedy algorithm
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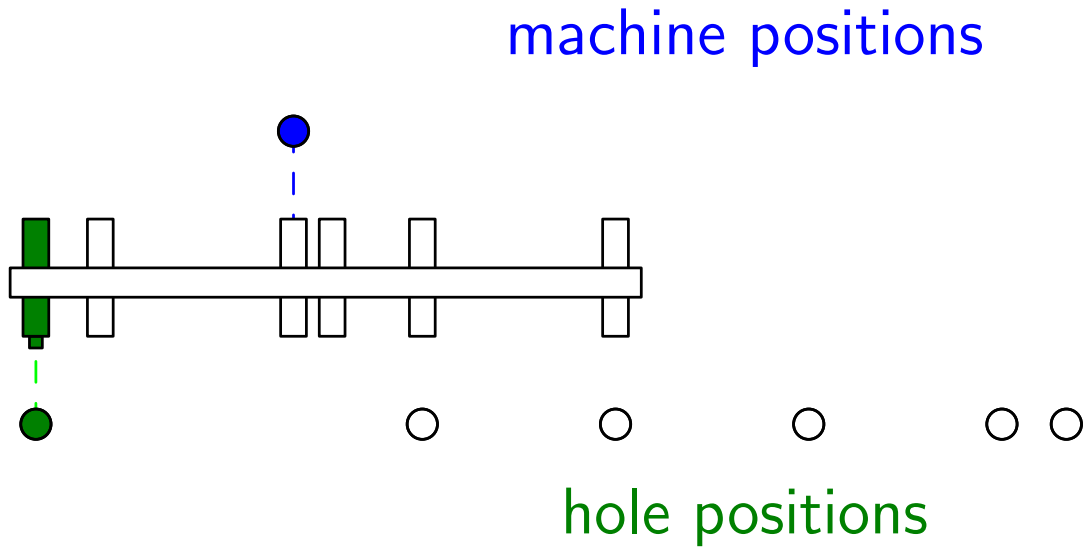

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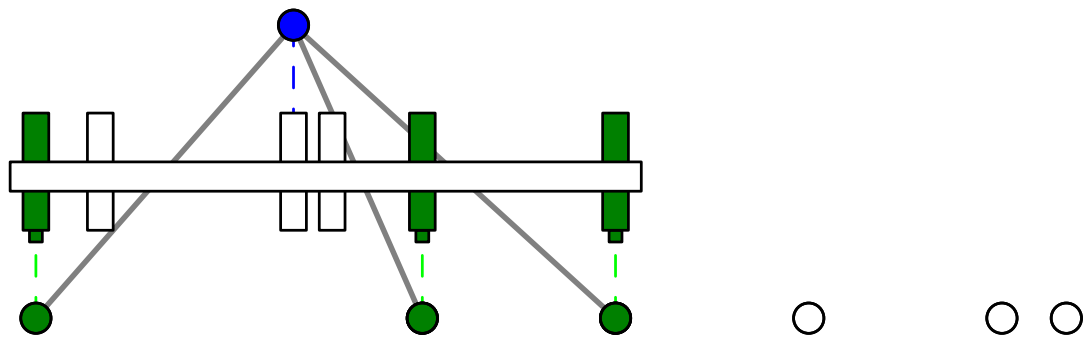
machine positions



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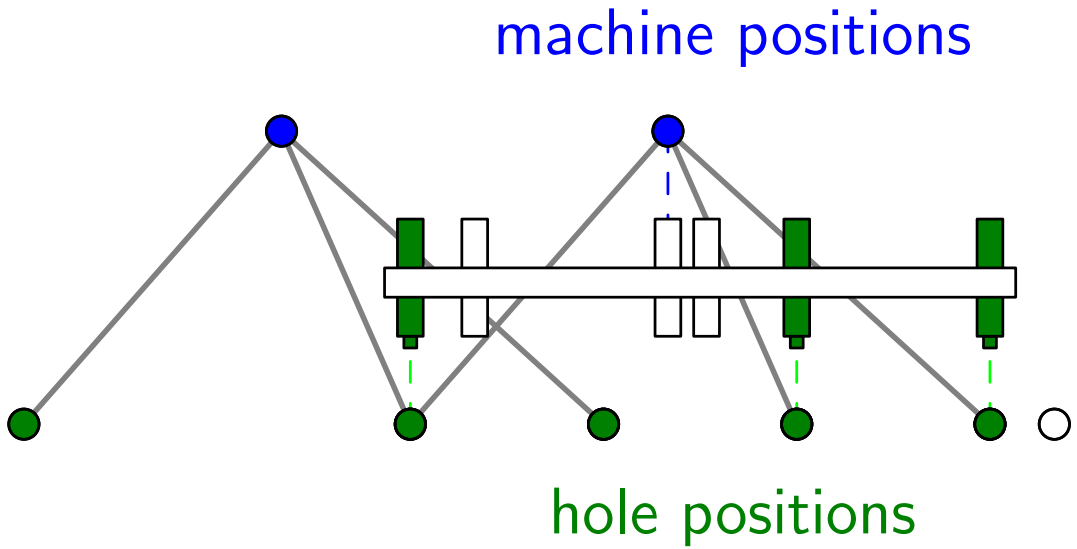

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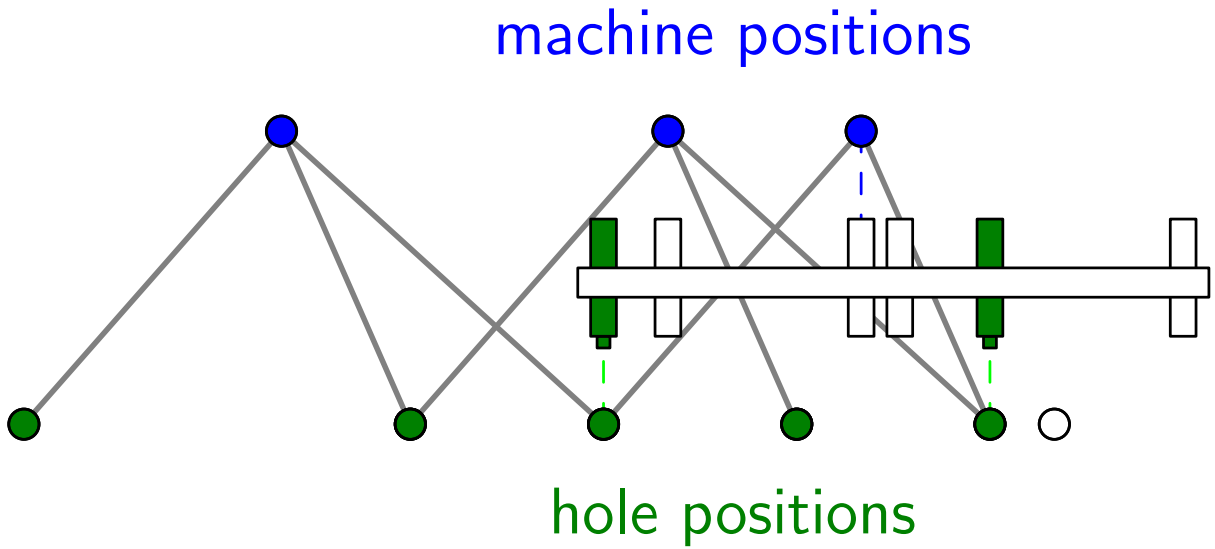

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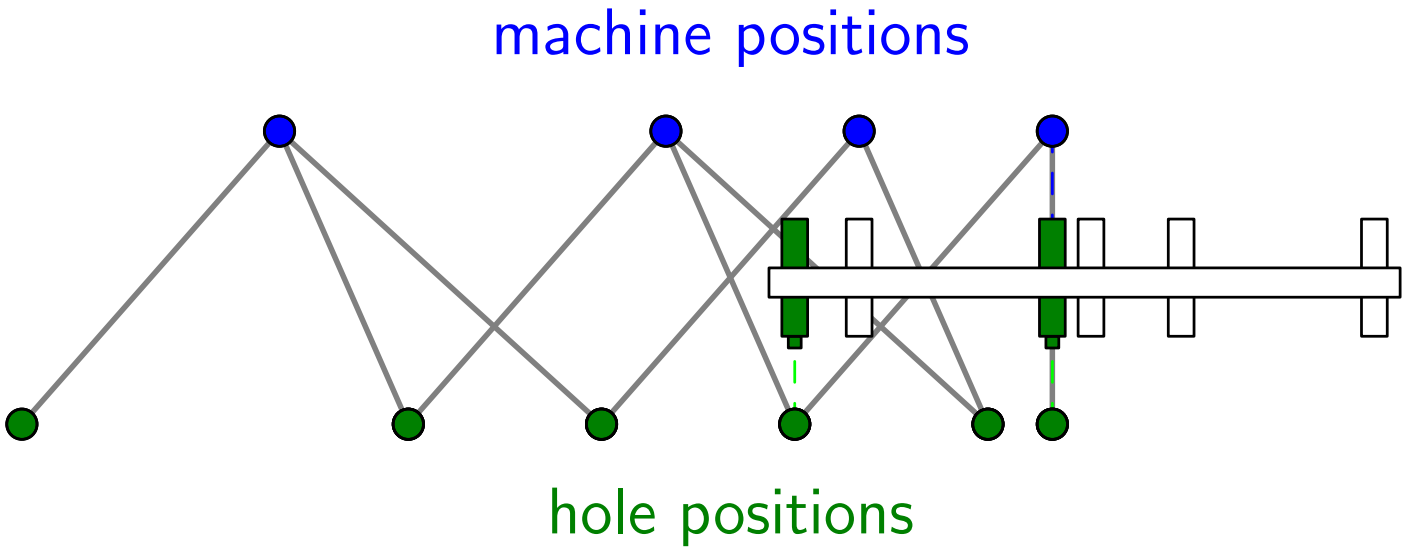

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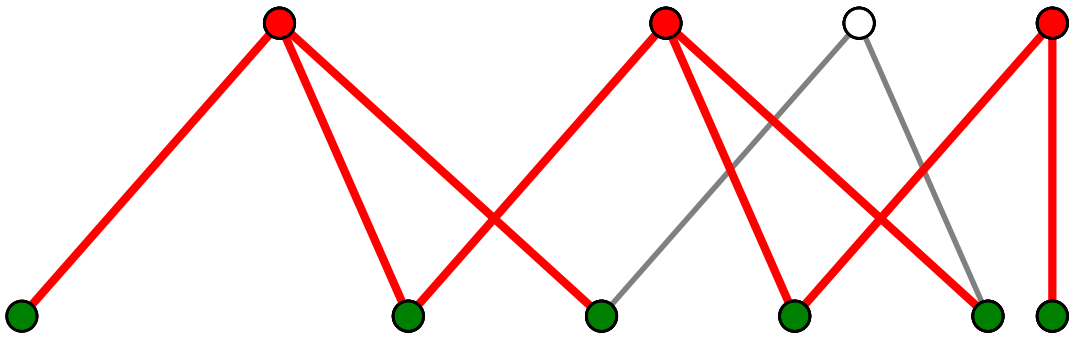
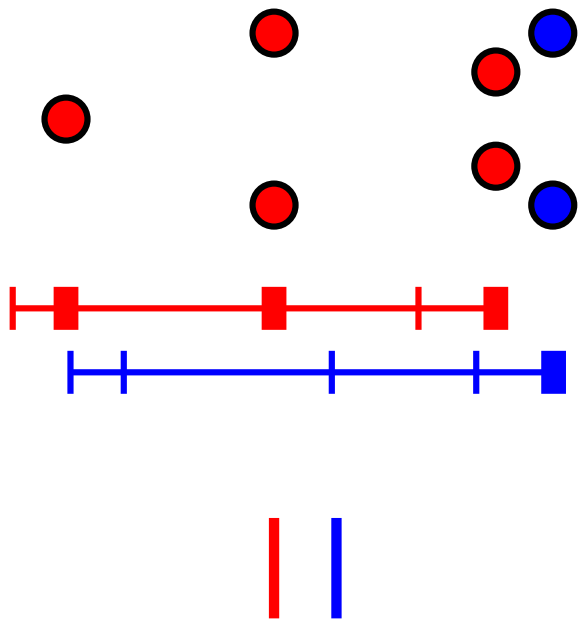
cover of holes by machine positions



hole positions

Algorithm: local search for an improved solution

- combine punching steps
- change stamp positions and assignments
- exchange order of punching steps



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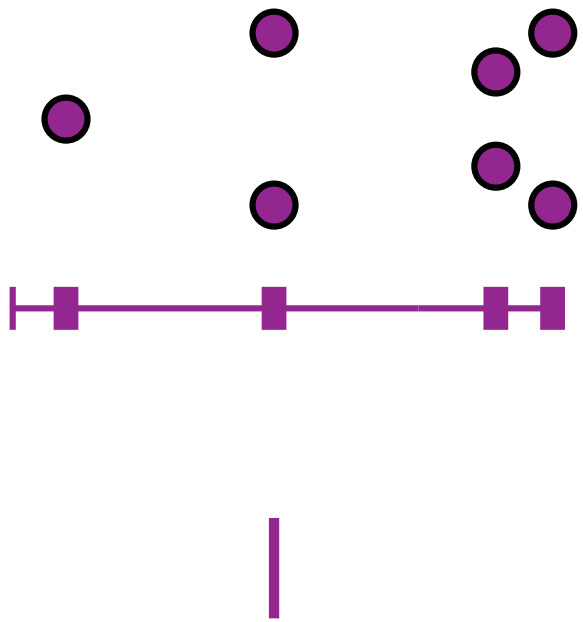

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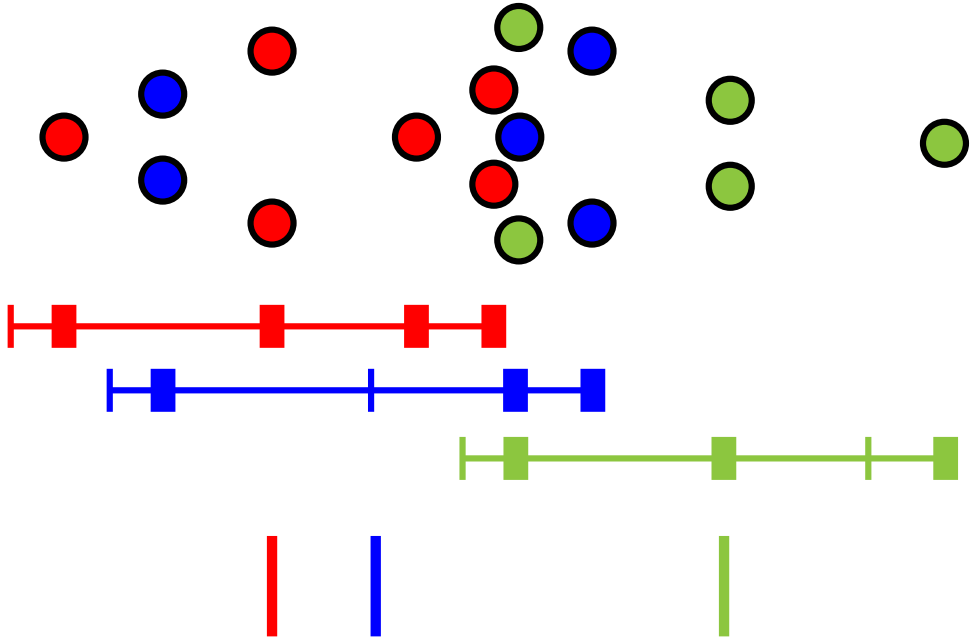

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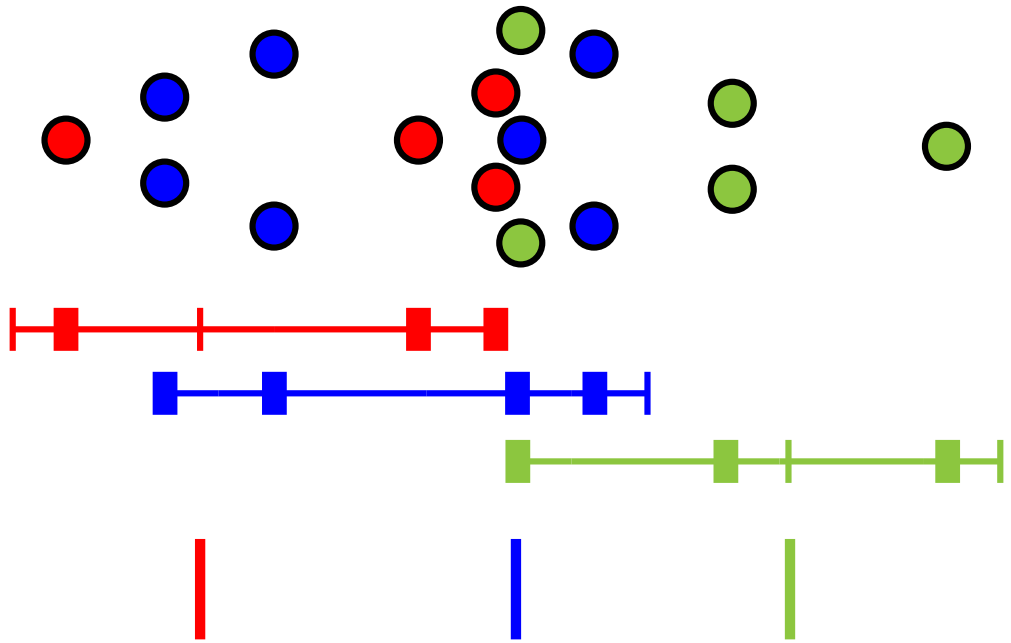
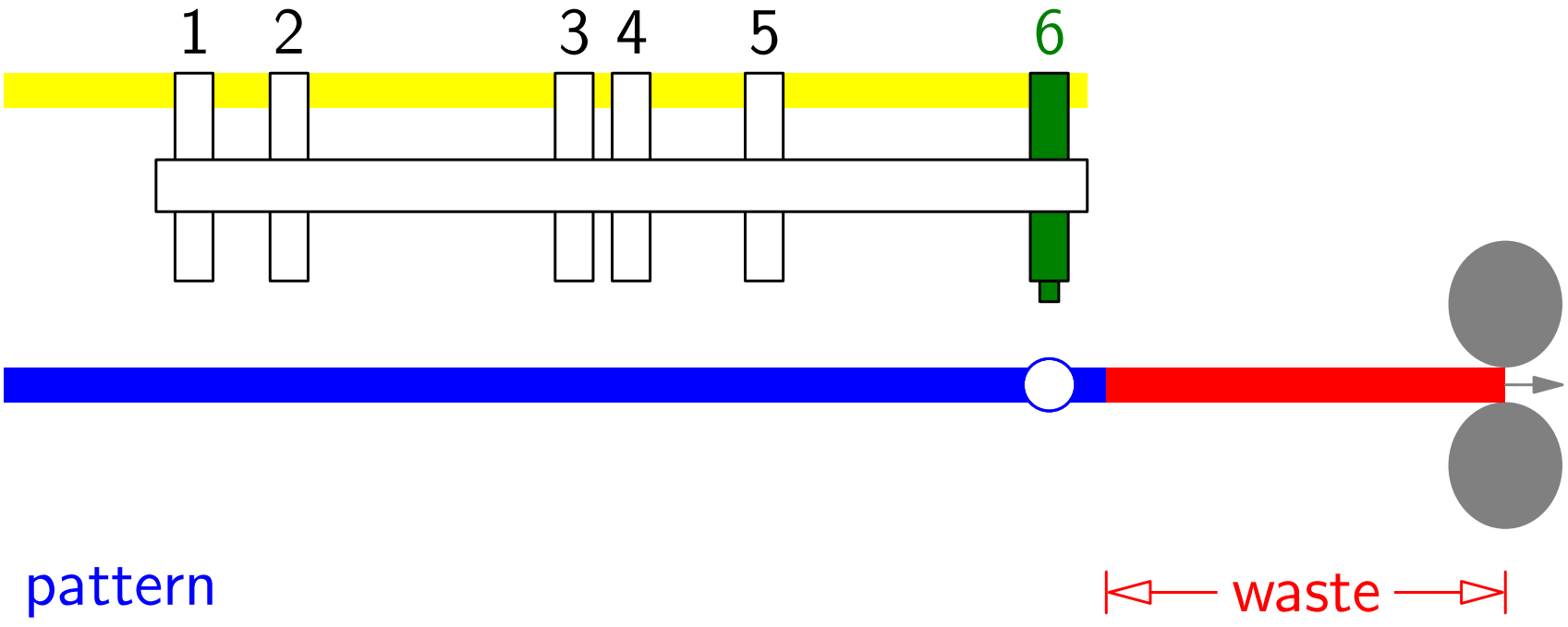


Waste

waste results from used stamp and its position at first step



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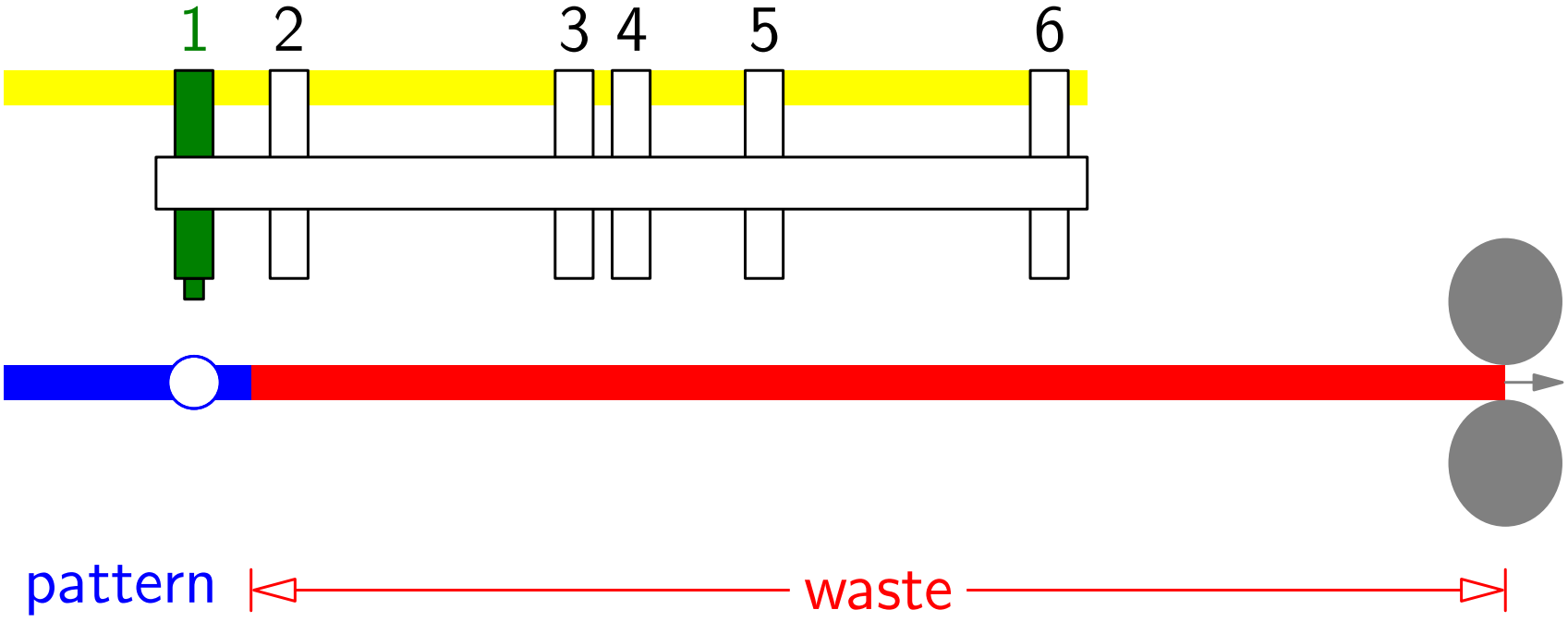

Evaluation

- evaluated using 250 given test instances
- solution helps saving both time and material
- in production use in Austria, Poland, Iran, and India

Thank you for your attention!

Jan Pöschko

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